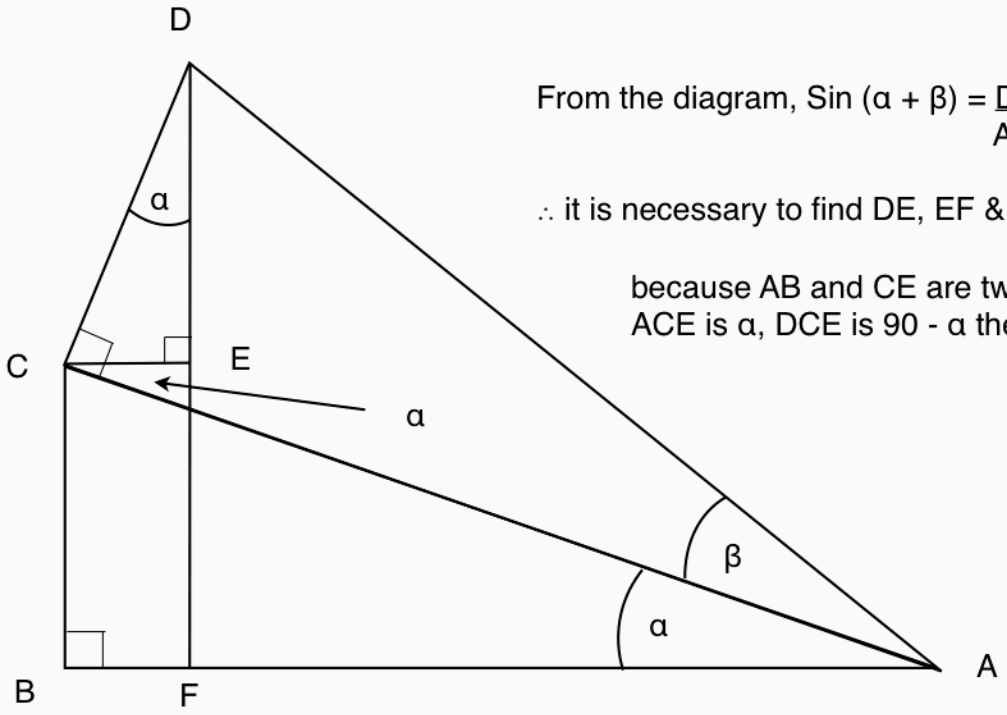
To prove Sin (α + β) = Sin α Cos β + Cos α Sin β



From the diagram, Sin (α + β) = $\frac{DF}{AD} = \frac{EF + DE}{AD}$

∴ it is necessary to find DE, EF & AD in terms of α & β

because AB and CE are two parallel lines, angle ACE is α, DCE is 90 - α therefore CDE is α.

In triangle CDE, Cos α = $\frac{DE}{CD}$ ∴ DE = CD Cos α ---- ①

In triangle ACD, Sin β = $\frac{CD}{AD}$ ∴ CD = AD Sin β ---- ②

Substituting ② into ① we get:- DE = AD Cos α Sin β ---- ③

In triangle ABC, Sin α = $\frac{BC}{AC}$ ∴ BC = AC Sin α ---- ④

In triangle ACD, Cos β = $\frac{AC}{AD}$ ∴ AC = AD Cos β ---- ⑤

Substituting ⑤ into ④ we get:- BC = AD Sin α Cos β ---- ⑥

From the diagram, Sin (α + β) = $\frac{EF + DE}{AD}$ and EF = BC ∴ Sin (α + β) = $\frac{BC + DE}{AD}$ ---- ⑦

Substituting ③ and ⑥ into ⑦ and cancelling out terms common to top and bottom of the equation we get:-

$$\text{Sin } (\alpha + \beta) = \frac{\cancel{AD}\text{ Sin } \alpha \text{ Cos } \beta + \cancel{AD}\text{ Cos } \alpha \text{ Sin } \beta}{\cancel{AD}}$$

Sin (α + β) = Sin α Cos β + Cos α Sin β QED