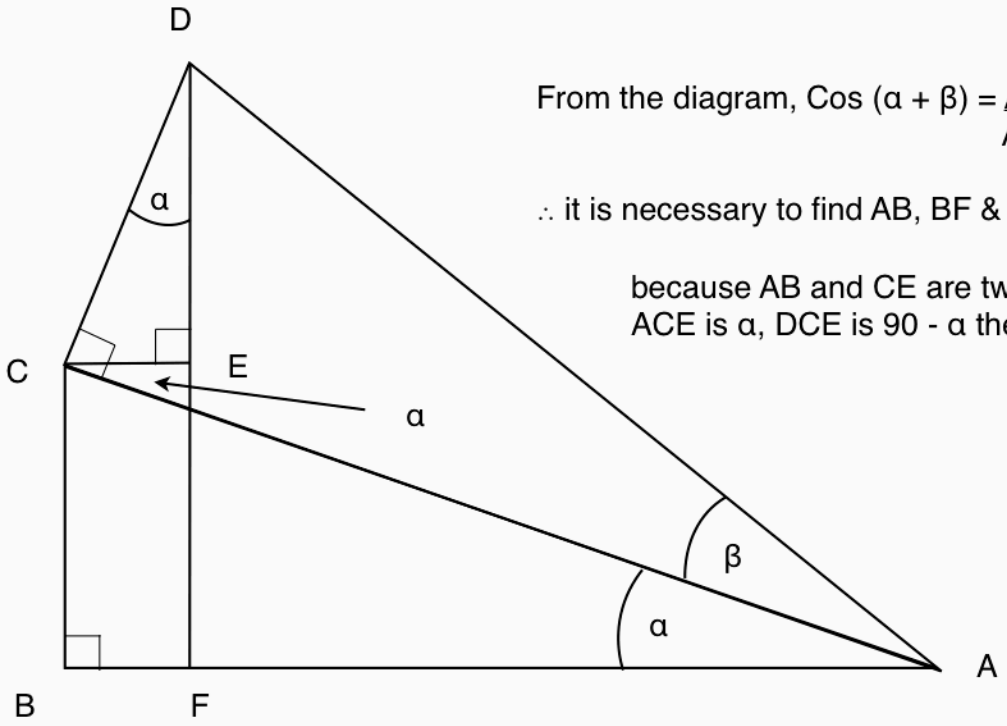
To prove Cos (α + β) = Cos α Cos β - Sin α Sin β



From the diagram, $\text{Cos}(\alpha + \beta) = \frac{AF}{AD} = \frac{AB - BF}{AD}$

∴ it is necessary to find AB, BF & AD in terms of α & β

because AB and CE are two parallel lines, angle ACE is α, DCE is 90 - α therefore CDE is α.

In triangle CDE, $\text{Sin}\ \alpha = \frac{CE}{CD}$ ∴ CE = CD Sin α ---- ①

In triangle ACD, $\text{Sin}\ \beta = \frac{CD}{AD}$ ∴ CD = AD Sin β ---- ②

Substituting ② into ① we get:- CE = AD Sin α Sin β ---- ③

In triangle ABC, $\text{Cos}\ \alpha = \frac{AB}{AC}$ ∴ AB = AC Cos α ---- ④

In triangle ACD, $\text{Cos}\ \beta = \frac{AC}{AD}$ ∴ AC = AD Cos β ---- ⑤

Substituting ⑤ into ④ we get:- AB = AD Cos α Cos β ---- ⑥

From the diagram, $\text{Cos}(\alpha + \beta) = \frac{AB - BF}{AD}$ and CE = BF ∴ $\text{Cos}(\alpha + \beta) = \frac{AB - CE}{AD}$ ---- ⑦

Substituting ③ and ⑥ into ⑦ and cancelling out terms common to top and bottom of the equation we get:-

$$\text{Cos}(\alpha + \beta) = \frac{\cancel{AD}\ \text{Cos}\ \alpha\ \text{Cos}\ \beta - \cancel{AD}\ \text{Sin}\ \alpha\ \text{Sin}\ \beta}{\cancel{AD}}$$

Cos (α + β) = Cos α Cos β - Sin α Sin β QED