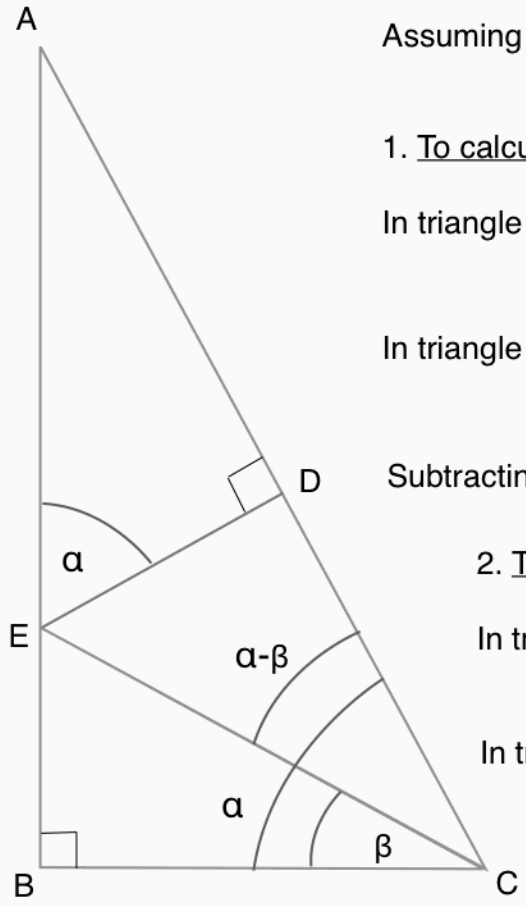
**To Prove Sin(α - β) = Sin α Cos β - Cos α Sin β**




Assuming the hypotenuse AC = 1

1. To calculate the length of AE

In triangle ABC, $\sin\alpha = \frac{AB}{1}$ ∴ AB = Sin α --- ①

In triangle BCE, $\sin\beta = \frac{EB}{EC}$ ∴ EB = EC Sin β --- ②

Subtracting ② from ① ∴ AE = Sin α - EC Sin β --- ③

2. To calculate the length of EC

In triangle ABC, $\cos\alpha = \frac{BC}{1}$ ∴ BC = Cos α --- ④

In triangle BCE, $\cos\beta = \frac{BC}{EC}$ ∴ $EC = \frac{BC}{\cos\beta}$ --- ⑤

Substituting ④ into ⑤ ∴ $EC = \frac{\cos\alpha}{\cos\beta}$ --- ⑥

3. To calculate the length of ED

In triangle ADE, $\cos\alpha = \frac{ED}{AE}$ ∴ ED = AE Cos α --- ⑦

Substituting ③ into ⑦ ∴ ED = (Sin α - EC Sin β) Cos α --- ⑧

4. To find the solution for Sin(α - β) using triangle CDE and known values of EC & ED

In triangle CDE, $\sin(\alpha - \beta) = \frac{ED}{EC}$ ∴ Substituting from ⑥ & ⑧

$$\sin(\alpha - \beta) = (\sin\alpha - EC \sin\beta)\cos\alpha \times \frac{\cos\beta}{\cos\alpha}$$

$$\sin(\alpha - \beta) = (\sin\alpha - \frac{\cos\alpha}{\cos\beta}\sin\beta)\ \cancel{\cos\alpha} \times \frac{\cos\beta}{\cancel{\cos\alpha}}$$

$$\sin(\alpha - \beta) = (\sin\alpha - \frac{\cos\alpha}{\cos\beta}\sin\beta)\cos\beta$$

Multiplying out and resolving the RHS of the equation:-

**Sin(α - β) = Sin α Cos β - Cos α Sin β** QED