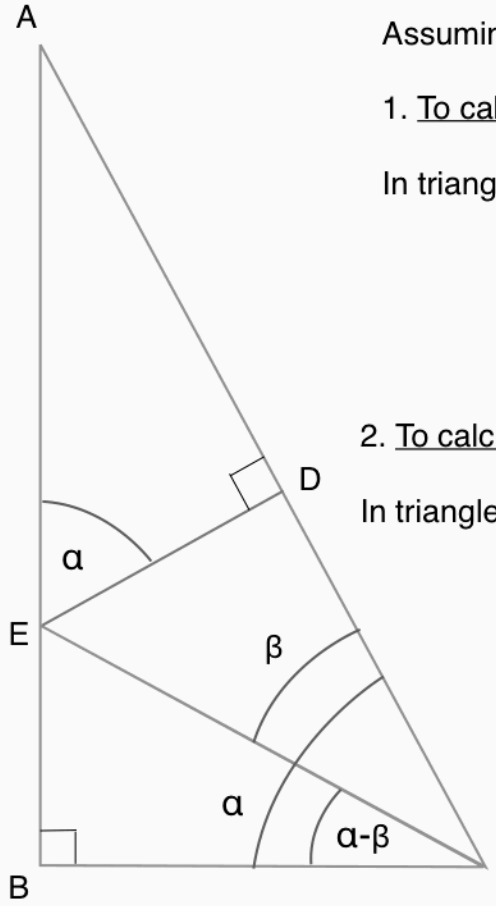
To Prove Cos(α - β) = Cos α Cos β + Sin α Sin β



Assuming the hypotenuse of triangle ADE, AE = 1

1. To calculate the length of ED & AD

In triangle ADE, $\cos \alpha = \frac{ED}{1}$ ∴ ED = Cos α --- ①

$\sin \alpha = \frac{AD}{1}$ ∴ AD = Sin α --- ②

2. To calculate the length of DC & EC

In triangle EDC, $\cos \beta = \frac{DC}{EC}$ ∴ DC = EC Cos β --- ③

$\sin \beta = \frac{ED}{EC}$ ∴ $EC = \frac{ED}{\sin \beta}$ --- ④

Substituting ① into ④ ∴ $EC = \frac{\cos \alpha}{\sin \beta}$ --- ⑤

3. To calculate the length of BC

In triangle ABC, $\cos \alpha = \frac{BC}{AC} = \frac{BC}{DC + AD}$ ∴ BC = Cos α (DC + AD) --- ⑥

Substituting ② & ③ into ⑥ BC = Cos α (EC Cos β + Sin α) --- ⑦

Substituting ⑤ into ⑦ $BC = \text{Cos } \alpha \left(\frac{\cos \alpha \cos \beta}{\sin \beta} + \text{Sin } \alpha\right)$ --- ⑧

4. To find the solution for Cos (α - β) using triangle BCE and known values of BC & EC

In triangle BCE, $\cos (\alpha - \beta) = \frac{BC}{EC}$ --- ⑨

∴ Substituting ⑤ and ⑧ into ⑨, $\cos (\alpha - \beta) = \frac{\cancel{\cos \alpha} \left(\frac{\cos \alpha \cos \beta}{\sin \beta} + \sin \alpha\right) \cdot \sin \beta}{\cancel{\cos \alpha}}$

Multiplying out and resolving the RHS of the equation:-

$$\cos (\alpha - \beta) = \frac{\cancel{\sin \beta} \cos \alpha \cos \beta}{\cancel{\sin \beta}} + \sin \alpha \sin \beta$$

Cos (α - β) = Cos α Cos β + Sin α Sin β QED