To derive equations for Tan (α ± β)
using the solutions for Sin (α ± β) and Cos (α ± β) already obtained

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \quad \text{-------} \ ①$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \quad \text{-------} \ ②$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \quad \text{-------} \ ③$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \quad \text{-------} \ ④$$

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1. $\tan(\alpha + \beta) = \dfrac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \quad \therefore \tan(\alpha + \beta) = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$

dividing the terms on top and bottom of the equation by Cos α Cos β and resolving:-

$$\tan(\alpha + \beta) = \frac{\sin\alpha \cancel{\cos\beta}/\cos\alpha \cancel{\cos\beta} + \cancel{\cos\alpha}\sin\beta/\cancel{\cos\alpha}\cos\beta}{\cancel{\cos\alpha\cos\beta}/\cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta/\cos\alpha\cos\beta}$$

$$\therefore \mathbf{\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}} \qquad \mathbf{QED}$$

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2. $\tan(\alpha - \beta) = \dfrac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} \quad \therefore \tan(\alpha - \beta) = \dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$

dividing the terms on top and bottom of the equation by Cos α Cos β and resolving:-

$$\tan(\alpha - \beta) = \frac{\sin\alpha \cancel{\cos\beta}/\cos\alpha \cancel{\cos\beta} - \cancel{\cos\alpha}\sin\beta/\cancel{\cos\alpha}\cos\beta}{\cancel{\cos\alpha\cos\beta}/\cancel{\cos\alpha\cos\beta} + \sin\alpha\sin\beta/\cos\alpha\cos\beta}$$

$$\therefore \mathbf{\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}} \qquad \mathbf{QED}$$